Fluctuating Twistor-Beam Solutions and Holographic Pre-Quantum Kerr-Schild Geometry.

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Abstract. Kerr-Schild (KS) geometry is based on a congruence of twistor null lines which forms a holographic space-time determined by the Kerr theorem. We describe in details integration of the non-stationary Debney-Kerr-Schild equations for electromagnetic excitations of black-holes taking into account the consistent back-reaction to metric. The exact KS solutions have the form of singular beam-like pulses supported on twistor null lines of the Kerr congruence. These twistor-beam pulses have very strong back reaction to metric and BH horizon and produce a fluctuating holographic KS geometry which takes an intermediate position between the Classical and Quantum gravity.

1. Introduction

Singular pp-waves [1], playing the role of plane waves in gravity, differ drastically from plane waves in flat spacetime, which is one of the origin of incompatibility of the Quantum theory and Gravity. Quantum theory works basically in momentum space, while Gravity demands explicit representation in configuration space-time. Twistor theory forms a bridge between them. Geometrically, twistor is a null line formed by a pair (x^μ, θ^α), where θ^α is a two-component spinor joined to the point $x^\mu \in M^4$. This spinor fixes a null direction $\sigma^\mu_{\dot\alpha\alpha}\bar\theta^{\dot\alpha}\theta^\alpha$ corresponding to momentum p^μ of a massless particle. A plane wave in twistor coordinates $T^I = \{\theta^\alpha, \mu_{\dot\alpha}\}, \quad \mu_{\dot\alpha} = x_\nu\sigma^\nu_{\dot\alpha\alpha}\theta^\alpha$ has the form $\exp\{ix_\mu\sigma^\mu_{\dot\alpha\alpha}\bar\theta^{\dot\alpha}\theta^\alpha\}$ and may be transformed to twistor space by a 'twistor' Fourier transform. The result, [2](p.2.5), corresponds to a singular beam in direction p^μ supported on the twistor null line with coordinates $T^I = \{\theta^\alpha, \mu_{\dot\alpha}\}$. Such twistor-beams are the sources of singular pp-wave solutions in gravity, and similar twistor-beams are obtained for typical exact electromagnetic (em) excitations of black-holes (BH) [3] presented in the KS class of the algebraically special solutions [4] which cover a wide range of the rotating and non-rotating BH's and cosmological solutions. Appearance of the twistor-beams in the exact solutions is not accidental, since twistors form a skeleton of the KS space-time.

In this work we give a detailed description of integration of the Debney-Kerr-Schild equations [4] which describe the exact em excitations of the Kerr black-hole and corresponding back reaction to metric[1] consistent with the Einstein-Maxwell equations with averaged stress-energy tensor.

The KS geometry has twosheeted holographic twistor structure which turns out to be perfectly adapted to quantum treatment suggested in [5, 6], and we show that the resulting set of the

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[1] In this treatment we neglect by the recoil of the excitations on the position of the BH.

solutions forms a fluctuating pre-quantum geometry taking intermediate position between the Classical and Quantum gravity.

2. Twistor structure of the KS geometry

The KS solutions are based on the KS form of metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \tag{1}$$

in which $\eta_{\mu\nu}$ is metric of auxiliary Minkowski space-time M^4 and k_μ is a field of null directions, forming a principal null congruence (PNC) $\mathcal{K}$. The BH solutions may be represented in two different forms $g_{\mu\nu}^\pm = \eta_{\mu\nu} + 2Hk_\mu^\pm k_\nu^\pm$, were $k^{\mu\pm}(x)$, $(x = x^\mu \in M^4)$ correspond to two different vector fields tangent to different congruences $\mathcal{K}^\pm$. The directions $k_\mu^\pm$ are determined in the Cartesian null coordinates $u = (z-t)/\sqrt{2}$, $v = (z+t)/\sqrt{2}$, $\zeta = (x+iy)/\sqrt{2}$, $\bar\zeta = (x-iy)/\sqrt{2}$ by the form

$$k_\mu^{(\pm)}dx^\mu = P^{-1}(du + \bar{Y}^\pm d\zeta + Y^\pm d\bar\zeta - Y^\pm \bar{Y}^{(\pm)}dv) \tag{2}$$

which depends on the complex angular coordinate $Y = e^{i\phi}\tan\frac{\theta}{2}$. Two solutions $Y^\pm(x)$ are determined by the **Kerr Theorem** [1, 4, 7, 8] which states that the (necessary for solutions of type D) geodesic and shear-free null congruences in M^4 are generated by algebraic equation $F = 0$, where $F(Z^p)$ is an arbitrary holomorphic function of the projective twistor coordinates.[2]

The Kerr congruence represents a vortex of null lines, which covers the spacetime twice: in the form of ingoing ($k^{\mu-} \in \mathcal{K}^-$) and outgoing vector fields ($k^{\mu+} \in \mathcal{K}^+$). It forms two sheets of the KS geometry with two different metrics $g_{\mu\nu}^\pm$ on the same spacetime M^4.
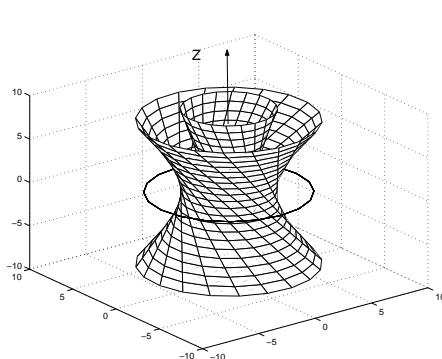

Figure 1. The Kerr singular ring and the Kerr congruence of twistor null lines.
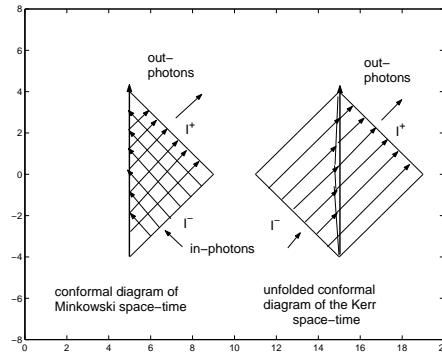
Figure 2. Penrose conformal diagrams. Unfolding of the auxiliary M^4 space of the Kerr spacetime yields twosheeted structure of a pre-quantum BH spacetime.

The em field has to be aligned with the rays of PNC and turns out to be different on the in- and out- sheets, and these two fields should not be mixed, which is ignored usually in the perturbative approach, leading to drastic discrepancy in the form of the fundamental solutions. The typical exact em solutions on the Kerr background have the form of singular twistor-beams propagating along the twistor null rays of the Kerr PNC, contrary to the smooth angular dependence of the typical wave solutions used perturbatively!

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[2] Projective twistor coordinates Z^p, $p = 1, 2, 3$ are related with twistor coordinates $\{\theta^\alpha,\ \mu_{\dot\alpha}\}$ as follows $T^I = \theta^1(1, Z^p)$.

3. Holographic KS structure

For a long time this twosheetedness was a mystery of the Kerr BH (see refs in [9]). It was suggested by Israel (1968) to truncate the second sheet and replace it by a rotating disk-like bubble covering the Kerr singular ring. Alternatively, the Kerr singular ring was considered as a closed 'Alice' string forming a gate to 'Alice' mirror world of the advanced fields [10]. Holographic approach resolves this problem unifying the both points of view: the source of Kerr solution has to be considered as a membrane separating the in- and out- parts of the KS space. It is supported by the membrane paradigm and by the obtained recently structure of the consistent Kerr-Newman source [11]. The both sheets of the KS space are necessary for description of quantum fields in gravity. In particular, Gibbons states in [5] that the curved spacetime $\mathcal{M}$ should be separated into two time-ordered regions $\mathcal{M}_-$ and $\mathcal{M}_+$ which are associated with ingoing and outgoing vacuum states $|0_- >$ and $|0_+ >$. If the source is absent, the KS basic solutions may be extended analytically from in- to out-sheet and vice versa. Presence of the source breaks this analyticity, separating the retarded and advanced fields, which allows one to consider the BH evaporation as a scattering of the in-vacuum on the BH-source. Similarly, in [6] authors consider a pre-quantum BH spacetime with separated the in- and out- sheets which correspond to a holographic correspondence: the source forms a holographically dual boundary (membrane) separating the in- and out- regions.[3] The usual Penrose conformal diagram, containing the in- and out-fields on the same of M^4, has to be unfolded in the holographic interpretation to split twosheetedness, as it is shown on the Fig.2. The null rays of Kerr congruence perform the lightlike projection of the past null infinity I^-, so the Kerr source appears as a holographic image of the data on the past null infinity I^-.

Conformal structure of the KN solution is determined by complex function $Y(x)$. Tetrad derivatives $\partial_a = e_a^\mu \partial_\mu$ of the function $Y(x)$ determine principal parameters of the KS holographic projection. In particular, function $Y(x)$ for the shear-free and geodesic congruences of the Kerr theorem has to satisfy the conditions

$$Y_{,2} = Y_{,4} = 0, \tag{3}$$

which show that the expression $dY = Y_{,a}\, e^a = Y_{,1}\, e^1 + Y_{,3}\, e^3$ is gradient of the complex null surfaces $Y = const.$, which form a fiber bundle of the KS space-time with fibers spanned by the tetrad forms e^1 and e^3, providing conformal properties of the KS holographic projection.

4. Stationary twistor-beam KS solutions

For the Kerr congruence function $F(Y, x)$ of the Kerr theorem is to be quadratic in Y, and the eq. $F = 0$ may resolved in explicit form. The corresponding solutions $Y(x)$ determine the stationary Kerr congruence via (2) and the adapted (oblate spheroidal) coordinate system r, θ, ϕ in which

$$Y = e^{i\phi} \tan \frac{\theta}{2}. \tag{4}$$

Function $F(Y, x)$ determines also the function H up to an arbitrary function $\psi(Y)$, and therefore, the KS metric (1). In agreement with [4]

$$H = \frac{1}{2}[m(Z + \bar{Z})P^{-1} - |\psi|^2 Z\bar{Z}P^{-2}], \tag{5}$$

where $Z = Y_{,1}$ and $(Z/P)^{-1} = -dF/dY$. Complex function $Z = \rho + i\omega$ characterizes the conformal properties of the congruence: expansion ρ and rotation ω of the projected image. For the Kerr-Newman BH solution at rest, Z is inversely proportional to a complex radial distance

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[3] In the model [11] it is a domain wall boundary of a rotating disk-like source forming a superconducting bubble.

$Z = -P/(r + ia \cos \theta)$ where function P is an extra conformal factor determined by Killing vector of the solution [4].

In particular, for the Kerr and Kerr-Newman (KN) geometries at rest

$$H = \frac{mr - |\psi|^2/2}{r^2 + a^2 \cos^2 \theta} \ .$$ (6)

The KN electromagnetic field is determined by the vector potential

$$\alpha = \alpha_\mu dx^\mu = -\frac{1}{2} Re \ [(\frac{\psi}{r + ia \cos \theta})e^3 + \chi d\bar{Y}],$$ (7)

where $\chi = 2 \int (1 + Y\bar{Y})^{-2} \psi dY$, which obeys the alignment condition $\alpha_\mu k^\mu = 0$.

For the Kerr solution m is mass of BH and $\psi = 0$. For the KN solution $\psi = q = const.$. However, any nonconstant holomorphic function $\psi(Y)$ yields also an exact KS solution, [4]. On the other hand, any nonconstant holomorphic functions on sphere acquire at least one pole. A single pole at $Y = Y_i$, $\psi_i(Y) = q_i/(Y - Y_i)$ produces the beam in angular directions $Y_i = e^{i\phi_i} \tan \frac{\theta_i}{2}$. The function $\psi(Y)$ acts immediately on the function H which determines the metric and position of the horizon. The given in [12] analysis showed that electromagnetic beams have very strong back reaction to metric and deform topologically the horizon, forming in the horizon the holes which allow matter to escape interior. For $\psi(Y) = \sum_i \frac{q_i}{Y - Y_i}$, the exact solutions have several beams in angular directions $Y_i = e^{i\phi_i} \tan \frac{\theta_i}{2}$, leading to the horizon with many holes. In far zone the twistor beams tend to the known exact singular pp-wave solutions [1].

5. Pre-quantum fluctuating KS geometry

The stationary KS beamlike solutions may be generalized to time-dependent wave pulses, [3]. Since the horizon is extra sensitive to electromagnetic excitations, it may also be sensitive to the vacuum electromagnetic field, and the vacuum beam pulses may produce a fine-grained structure of fluctuating microholes in the horizon, allowing radiation to escape interior of black-hole.
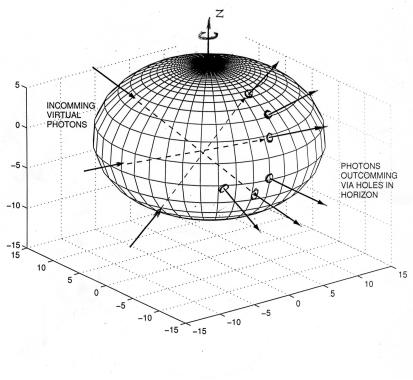

Figure 3. Twistor-beam pulses perforate the BH horizon forming its fluctuating fine-grained structure.

Twistor-beam pulses depend on a retarded time τ and have to obey to the non-stationary Debney-Kerr-Schild (DKS) equations [4]. The corresponding solutions acquire an extra radiative term $\gamma(Y, \tau)$. The long-term attack on the DKS equations [3], has led to the obtained time-dependent solutions of the fluctuating Kerr-Schild space-times. Derivation of the time-dependent solutions will be given in the next section. Here we give a brief preliminary description of the general structure of the DKS solutions obtained by integration in [4].

The general em field is described by the self-dual tetrad components,

$$\mathcal{F}_{12} = AZ^2, \quad \mathcal{F}_{31} = \gamma Z - (AZ)_{,1}, \tag{8}$$

where $\mathcal{F}_{ab} = e_a^\mu e_b^\nu \mathcal{F}_{\mu\nu}$. We will assume that the mass of BH is much greater then the energy of the BH excitation. It allows us to neglect by recoil and consider the Kerr congruence as stationary. Fluctuation of the metric will be related with fluctuation of the em field. By these assumptions we obtain, [3], that the exact time-dependent solutions describe the em radiation from BH which contains two components:

a) a set of the singular beam pulses (determined by function $\psi(Y,\tau)$) propagating along the Kerr PNC and breaking the topology and impenetrability of the horizon; and

b)the regularized radiative component (determined by $\gamma_{reg}(Y,\tau)$) which is smooth and determines evaporation of the black-hole.

The obtained solutions describe excitations of singular electromagnetic beams $A = \psi(Y,\tau)/P^2$ on the Kerr-Schild background and the back reaction of the singular field $\psi(Y,\tau)$ on the metric leading to fine-grained fluctuations of the metric and black-hole horizon.

The most essential difference from the stationary case is the appearance of an extra field $\gamma(Y,\tau)$. This term is absent in the expression for (6), and therefore, it does not have immediate action on the KS metric. One sees also from (8) that it generates the null EM radiation along the Kerr congruence which fall off asymptotically as $\sim Z = P/(r + ia\cos\theta) \sim 1/r$, and so, it is the leading term at infinity. The corresponding components of the stress-energy tensor

$$T_{\mu\nu} = T_{33} e_\mu^3 e_\nu^3 \sim = \frac{1}{2}\bar{\gamma}\gamma k_\mu k_\nu \tag{9}$$

describe a flow of energy along the Kerr congruence and are known as radiation of the Vaidya 'shining star' solution [1]. This term appears also in the Kinnersley 'photon rocket' solution [1], where it is related with acceleration of the source.

It shows explicitly that the horizon turns out to be covered by fluctuating micro-holes which make it penetrable for outgoing radiation. In the same time, the BH radiation is determined by the smooth field $\gamma_{reg}(Y,\tau)$, corresponding to the Vaidya "shining star" radiation. The holographic space-time forms a fluctuating twosheeted pre-geometry which reflects the dynamics of singular beam pulses. This pre-geometry is classical, but has to be still regularized to get the usual smooth classical space-time. In this sense, it takes an intermediate position between the classical and quantum gravity.

Note also that this structure may be generalized by using the Kerr theorem with the higher degree in Y functions $F(Y)$, which leads to the multi-particle KS solutions [13] having the complicate networks of twistor-beams.

6. Time-dependent Kerr-Schild solutions

The nearest time-dependent generalization of (4) is given by the form

$$\psi(Y,\tau) = \sum_i c_i(\tau)(Y - Y_i)^{-1}, \tag{10}$$

where one assumes that an elementary beam has $c_i(\tau) = q_i(\tau)e^{i\omega_i\tau}$ where $q_i(\tau)$ is amplitude and ω_i is a carrier frequency. Similar to stationary case, the non-stationary Kerr-Schild solutions contain the singular beams which change the structure of black hole horizon, but the beams may acquire the form of time-dependent pulses. Since the mass of a black hole is much more than the energy of its excitation, we will neglect by recoil and take the assumption that the black hole is at rest and the Kerr congruence remains undisturbed. It corresponds to $P = 2^{-1/2}(1 + Y\bar{Y})$ and $\dot{P} = 0$.

The obtained in [4] DKS general equations are the equations for functions A and γ which determine electromagnetic field

$$A_{,2} - 2Z^{-1}\bar{Z}Y_{,3}\,A = 0, \quad A_{,4} = 0, \tag{11}$$

$$\mathcal{D}A + \bar{Z}^{-1}\gamma_{,2} - Z^{-1}Y_{,3}\,\gamma = 0, \tag{12}$$

and the equations for function M which determine gravitational field taking into account the back reaction caused by electromagnetic field

$$M_{,2} - 3Z^{-1}\bar{Z}Y_{,3}\,M = A\bar{\gamma}\bar{Z}, \tag{13}$$

$$\mathcal{D}M = \frac{1}{2}\gamma\bar{\gamma}. \tag{14}$$

The used here operator $\mathcal{D}$ is

$$\mathcal{D} = \partial_3 - Z^{-1}Y_{,3}\,\partial_1 - \bar{Z}^{-1}\bar{Y}_{,3}\,\partial_2\ . \tag{15}$$

General equations for the nonstationary EM KS solutions were obtained by by Debney, Kerr and Schild [4] in 1968. However, in [4] the equations were fully integrated out only for the particular case $\gamma = 0$ which corresponds to stationary solutions.[4]

The first non-stationary wave EM solutions on the KS background were obtained in [14] and contained singular beams along the $\pm z-$ half-axis. There appeared the conjecture that there is no the exact regular time-dependent solutions at all, and for very seldom exclusions the exact solutions should have the singular beams.

The DKS equations are simplified by using the DKS relation

$$Y_{,3} = -ZP_{\bar{Y}}/P. \tag{16}$$

The equation (11) for function A takes the same form as in stationary case,

$$(AP^2)_{,2} = 0,\ \ A_{,4} = 0, \tag{17}$$

where the general solution is $A = \psi/P^2$, and function ψ has to obey

$$\psi_{,2} = \psi_{,4} = 0. \tag{18}$$

To obtain the non-stationary KS solutions we introduce a complex retarded-time parameter τ which satisfies the relations

$$(\tau)_{,2} = (\tau)_{,4} = 0\ . \tag{19}$$

It allows us to represent the equation (11) in the form $(AP^2)_{,2} = 0$, and to get solution which has the same form

$$A = \psi(Y,\tau)/P^2 \tag{20}$$

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[4] The degree of difficulty of this problem is described in the excerptions from the given by Roy Kerr in [15] (see Part 3 on the page 2481) review on the history of the Kerr solution. In this review Roy calls the eqs. (8), (17) and (18) as "easy" ones which *"... showed that the Einstein-Maxwell field depends on three functions, M, A, and γ ... restricted by "hard" field equations..."* [The "hard" equations are the EM eq. (12) and gravitational equation (13). AB.] ... *"I could not solve the later unless $\gamma = 0$ so I temporarily took this as an additional assumption and continued. This led to the complete charged Kerr-Schild metric metrics including, of course, charged Kerr. The null congruence for the later is the same as the Kerr congruence but the EM field depends on an arbitrary function of a complex variable...If it is a more complicated function then the EM field will probably be singular.* [This is exactly the case of stationary beam-like KS solutions. AB.] *At that point I turned the problem over to G.C. Debney ... to see if he could solve the problem when $\gamma \neq 0$. ... In the end it became clear that the tree of us could not solve the general problem."*

with the exception of the extra dependence of function ψ from the retarded-time parameter τ.

The principal difference from the stationary case is contained in the second electromagnetic DKS equation. Action of operator $\mathcal{D}$ on the variables $Y, \bar{Y}$ and τ is

$$\mathcal{D}Y = \mathcal{D}\bar{Y} = 0. \tag{21}$$

For the considered here case $P = 2^{-1/2}(1 + Y\bar{Y})$, and $\dot{P} = 0$, which yields

$$\mathcal{D}\tau = -1/P . \tag{22}$$

Using (16), (21) and (22), we reduce the equation (12) to the form

$$\dot{A} = -(\gamma P),_{\bar{Y}} , \tag{23}$$

where $(\dot{\ }) \equiv \partial_\tau$. Integrating this equation we obtain

$$\gamma = -P^{-1}\int \dot{A}d\bar{Y} = \frac{2^{1/2}\dot{\psi}}{P^2 Y} + \phi(Y,\tau)/P, \tag{24}$$

where ϕ is an arbitrary analytic function of Y and τ.

This solution shows that *any non-stationarity in electromagnetic field ($\dot{A} \neq 0$) generates an extra function γ* which, in accord with (8), generates also the lightlike electromagnetic radiation along the Kerr congruence. Such a radiation is well-known for the Vaidya 'shining star' solution [1], in which the field $A = \psi P^{-2}$ is absent and γ is incoherent, being related with the loss of total mass into radiation. The KS analog of this Vaidya relation is obtained from (14) by substitution

$$M = m/P^3, \tag{25}$$

and using (16), (21) and (22), and also the fact that $t = \frac{1}{2}(\tau + \bar{\tau})$,

$$\dot{m} = -\frac{1}{2}P^4 \gamma^{(reg)}\bar{\gamma}^{(reg)}. \tag{26}$$

It is one of two gravitational equations determining self-consistency of the Kerr-Schild solution [4]. The most important consequence following from DKS equations in the non-stationary case is the fact that the field γ appears inevitable, however it does not contribute to deformation of the horizon, since it is absent in the function H of (6). Its back reaction on the metric is smooth and circumstantial, acting only via the slowly decreasing mass parameter m. Therefore, in the non-stationary Kerr-Schild case we obtain that the *lightlike fields, determined by functions ψ and γ, have essentially different impact on the horizon.*

Function $\psi = \psi(\tau, Y)$ obeys the equation (18) which shows that the retarded time τ has to satisfy the conditions similar to (3), and therefore, gradient of τ is to be aligned to congruence,

$$k^\mu \tau,_\mu = 0. \tag{27}$$

It was obtained in [14] that the corresponding retarded-time parameter has the form

$$\tau = t - r - ia\cos\theta. \tag{28}$$

Since function ϕ contributes only to γ it does not impact on the form of the horizon too. Important role of this function is obtained from the analysis of the gravitational Kerr-Schild equation (13) which is reduced by using (16), (21) and (25) to the form

$$m,_{\bar{Y}} = \psi\bar{\gamma}P. \tag{29}$$

If we note that $\gamma \sim \dot{\psi} \approx i\omega\psi$, we obtain that the r.h.s. of this equation tends to zero in the low-frequency limit, as well as the r.h.s of the equation for $\dot{m}$. So, the full solution will tend to consistent with gravity at least in the low-frequency limit [9]. For the simplicity we consider first the case of a single time-dependent beam generated by a single pole at $Y = Y_i$. In this case

$$\dot{\psi}_i = \dot{c}_i(\tau)/(Y - Y_i) \tag{30}$$

and the solution (24) looks singular. However, we have still the free function ϕ, form of which and parameters (time-dependence and position of its pole) may be tuned to cancel the pole of function $\dot{\psi}_i$, and thus, regularize the solution. We define the analytic in Y function

$$P_i = P(Y, \bar{Y}_i) = 2^{-1/2}(1 + Y\bar{Y}_i) \tag{31}$$

and set

$$\phi_i^{(tun)}(Y, \tau) = -\frac{2^{1/2}\dot{c}_i(\tau)}{Y(Y - Y_i)P_i}. \tag{32}$$

One sees that the required analyticity of function $\phi_i^{(tun)}(Y, \tau)$ in Y is ensured. Using the equality

$$(P_i - P)/(Y_i - Y) = \frac{Y}{\sqrt{2}}\frac{(\bar{Y}_i - \bar{Y})}{(Y_i - Y)}, \tag{33}$$

we obtain that the regularized solution $\gamma_i^{(reg)} = \frac{2^{1/2}\dot{\psi}_i}{P^2Y} + \phi_i^{(tun)}(Y, \tau)/P$, takes the form

$$\gamma_i^{(reg)} = \frac{1}{P^2}\frac{\dot{c}_i}{P_i}[\frac{\bar{Y}_i - \bar{Y}}{Y_i - Y}]. \tag{34}$$

The r.h.s. of the equation (29) for this regular solution takes the form

$$\psi_i\bar{\gamma}_i^{(reg)}P = -\frac{c_i\dot{\bar{c}}_i}{P\bar{P}_i(\bar{Y} - \bar{Y}_i)}. \tag{35}$$

The equation (29) takes the form

$$m = -\int d\bar{Y}\frac{c_i\dot{\bar{c}}_i}{P\bar{P}_i(\bar{Y} - \bar{Y}_i)} \tag{36}$$

and may now be integrated by using the Cauchy integral formula, $\oint \frac{f(z)dz}{z-z'} = 2\pi i f(z')$, and we obtain the expression

$$m = m_0(Y) - 2\pi i\frac{c_i\dot{\bar{c}}_i}{P_iP_{ii}} , \tag{37}$$

containing free function $m_0(Y)$, and also the depending on Y contribution from the residue at singular point $\bar{Y}_i$, in which

$$P_{ii} = \frac{1}{\sqrt{2}}(1 + Y_i\bar{Y}_i) \tag{38}$$

is a constant. On the real slice P is real, $P_i \to P_{ii}$, and the real part of m takes the form

$$\Re e\ m = m_0 - i\pi\frac{c_i\dot{\bar{c}}_i - \dot{c}_i\bar{c}_i}{P_{ii}^2}, \tag{39}$$

where m_0 is a real constant. Imaginary part is

$$\Im m \; m = -i\pi \frac{c_i \dot{\bar{c}}_i + \dot{c}_i \bar{c}_i}{P_{ii}^2} \; . \tag{40}$$

If $c_i(\tau)$ is expressed via slowly varying amplitude $q_i(\tau)$ and the carrier frequency ω_i, $c_i(\tau) = q_i(\tau)e^{-i\omega_i\tau}$, the impact of the carrier frequency disappears and we obtain

$$\Re e \; m = m_0 - 2\pi\omega_i \frac{|q_i|^2}{P_{ii}^2} - i\pi \frac{q_i \dot{\bar{q}}_i - \dot{q}_i \bar{q}_i}{P_{ii}^2} \; , \tag{41}$$

So, the result of integration yields the time-dependence of m caused by the amplitude $q_i(\tau)$ and the weak and slow imaginary contribution

$$\Im m \; m = -i\pi \frac{q_i \dot{\bar{q}}_i + \dot{q}_i \bar{q}_i}{P_{ii}^2} \; ., \tag{42}$$

meaning of which is not clear at this stage. We have

$$m = m_0 - 2\pi\omega_i \frac{|q_i|^2}{P_{ii}^2} - 2\pi i \frac{q_i \dot{\bar{q}}_i}{P_{ii}^2} = m_0 - 2\pi i \frac{q_i}{P_{ii}^2}(\dot{\bar{q}}_i - i\omega_i \bar{q}_i) \; . \tag{43}$$

Influence of the extra contributions from $\dot{q}_i(\tau)$ falls off as $1/T$, where T is effective time of action of the beam. This contribution is negligible in the limit of the very long beams. Although these relations may be interpreted physically as a reaction of the mass parameter to single pulse, they cannot be considered literally, since the imaginary mass is inconsistent with the KS class of metrics. To provide consistency with gravitational sector we enforced to perform an averaging of the mass term over time, which is equivalent to averaging of the stress-energy tensor.

Further, it is known [1] that the gravitational equation (26) for radiative solutions is not equation really, but is only a definition of the loss of mass in radiation $\dot{m} = -\frac{1}{2}\frac{\dot{c}_i \dot{\bar{c}}_i}{|P_i|^2}$. On the real slice $P_i \to P_{ii}$, and in terms of the amplitudes of beams we obtain

$$\dot{m} = -\frac{1}{2}\frac{\omega_i^2 |q_i|^2 + |\dot{q}_i|^2}{P_{ii}^2} = -\frac{1}{2P_{ii}^2}(\dot{q}_i - i\omega_i q_i)(\dot{\bar{q}}_i - i\omega_i \bar{q}_i). \tag{44}$$

Similar to the Vaidya 'shining star' solution [1], for the case of the many stochastic pulses, we assume their frequencies as uncorrelated. There appears a double sum over the interacting beams and extra beating between them with fluctuations which drop out after averaging. The surviving radiation contains only slow fluctuations of the mass term caused by amplitudes of the self-correlated beams, i.e. the sum of the partial solutions pulses. Therefore, the obtained solutions turn out to be consistent with respect to the Einstein equations with averaged r.h.s., which for the DKS gravitational equations is equivalent to

$$m,_{\bar{Y}} = < P\psi\bar{\gamma}^{(reg)} >, \quad \dot{m} = -\frac{1}{2} < P^2\gamma^{(reg)}\bar{\gamma}^{(reg)} > \; . \tag{45}$$

The averaging removes the beating of the incoherent beams, however, it does not remove the sharp back-reaction of the beams to metric and horizon, caused by the poles in function $\psi(Y,\tau)$ in agreement with (6).

Therefore, the obtained solutions are exact and consistent with the Einstein-Maxwell system of equations with averaged stress-energy tensor. Recall also that we have used the restriction on the absence of recoil. The case of the non-zero recoil is very hard and was considered earlier by Kinnersley only for the non-rotating case [1]. It may be very important for the problem of scattering of spinning particles in twistor theory.

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